Monster = Group of Lattice Bosonic String Theory

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A physically realistic Lattice Bosonic String Theory (in which Strings are interpreted as World-Lines) containing gravity and the U(1)xSU(2)xSU(3) Standard Model can be constructed through a 12-Step process:

Step 1:

Consider the 26 Dimensions of Bosonic String Theory as the 26-dimensional traceless part J3(O)o

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 a        O+       Ov

 O+*      b        O-

 Ov*      O-*    -a-b
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(where Ov, O+, and O- are in Octonion space with basis {1,i,j,k,E,I,J,K} and a and b are real numbers with basis {1})

of the 27-dimensional Jordan algebra J3(O) of 3x3 Hermitian Octonion matrices.

Step 2:

Take a D3 brane to correspond to the Imaginary Quaternionic associative subspace spanned by {i,j,k} in the 8-dimenisonal Octonionic Ov space.

Step 3:

Compactify the 4-dimensional co-associative subspace spanned by {E,I,J,K} in the Octonionic Ov space as a CP2 = SU(3)/U(2), with its 4 world-brane scalars corresponding to the 4 covariant components of a Higgs scalar.

Add this subspace to D3, to get D7.

Step 4:

Orbifold the 1-dimensional Real subspace spanned by {1} in the Octonionic Ov space by the discrete multiplicative group Z2 = {-1,+1}, with its fixed points {-1,+1} corresponding to past and future time. This discretizes time steps and gets rid of the world-brane scalar corresponding to the subspace spanned by {1} in Ov. It also gives our brane a 2-level timelike structure, so that its past can connect to the future of a preceding brane and its future can connect to the past of a succeeding brane.

Add this subspace to D7, to get D8.

D8, our basic Brane, looks like two layers (past and future) of D7s.

Beyond D8 our String Theory has 26 - 8 = 18 dimensions, of which 25 - 8 have corresponding world-brane scalars:

- 8 world-brane scalars for Octonionic O+ space;
- 8 world-brane scalars for Octonionic O- space;
- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 5:

To get rid of the world-brane scalars corresponding to the Octonionic O+ space, orbifold it by the 16-element discrete multiplicative group Oct16 = {+/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K} to reduce O+ to 16 singular points {-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K}.

- Let the 8 O+ singular points {-1,-i,-j,-k,-E,-I,-J,-K} correspond to the fundamental fermion particles {neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark} located on the past D7 layer of D8.
- Let the 8 O+ singular points {+1,+i,+j,+k,+E,+I,+J,+K} correspond to the fundamental fermion particles {neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark} located on the future D7 layer of D8.

This gets rid of the 8 world-brane scalars corresponding to O+, and leaves:

- 8 world-brane scalars for Octonionic O- space;

- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 6:

To get rid of the world-brane scalars corresponding to the Octonionic O- space, orbifold it by the 16-element discrete multiplicative group Oct16 = {+/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K} to reduce O- to 16 singular points {-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K}.

- Let the 8 O- singular points {-1,-i,-j,-k,-E,-I,-J,-K} correspond to the fundamental fermion anti-particles {anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark} located on the past D7 layer of D8.
- Let the 8 O- singular points {+1,+i,+j,+k,+E,+I,+J,+K} correspond to the fundamental fermion anti-particles {anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark} located on the future D7 layer of D8.

This gets rid of the 8 world-brane scalars corresponding to O-, and leaves:

- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 7:

Let the 1 world-brane scalar for real a space correspond to a Bohm-type Quantum Potential acting on strings in the stack of D8 branes.

Interpret strings as world-lines in the Many-Worlds, short strings representing virtual particles and loops.

Step 8:

Fundamentally, physics is described on HyperDiamond Lattice structures.

There are 7 independent E8 lattices, each corresponding to one of the 7 imaginary octionions. They can be described as iE8, jE8, kE8, EE8, IE8, JE8, and KE8.

Further, an 8th naturally related, but dependent, E8 lattice corresponds to the real octonions and can be described as 1E8.

Give each D8 brane structure based on Planck-scale E8 lattices so that each D8 brane is a superposition/intersection/coincidence of the eight E8 lattices.

Step 9:

Since Polchinski says "... If r D-branes coincide ... there are r^2 vectors, forming the adjoint of a U(r) gauge group ...", make the following assignments:

- a gauge boson emanating from D8 only from its 1E8 lattice is a U(1) photon;
- a gauge boson emanating from D8 only from its 1E8 and EE8 lattices is a U(2) weak boson;
- a gauge boson emanating from D8 only from its IE8, JE8, and KE8 lattices is a U(3) gluon.

Step 10:

Since Polchinski says "... there will also be r^2 massless scalars from the components normal to the D-brane. ... the collectives coordinates ... X^u ... for the embedding of n D-branes in spacetime are now enlarged to nxn matrices. This 'noncummutative geometry' ...[may be]... an important hint about the nature of spacetime. ...", make the following assignment:

The 8x8 matrices for the collective coordinates linking a D8 brane to the next D8 brane in the stack are needed to connect the eight E8 lattices of the D8 brane to the eight E8 lattices of the next D8 brane in the stack.

We have now accounted for all the scalars.

Step 11:

Now describe gauge bosons emanating from D8 from its iE8, jE8, or kE8 lattices as a U(2,2) conformal gauge boson, so that we have:

- closed-string gravity for Penrose-Hameroff Quantum Consciousness and the Bohm Quantum Potential; and
- conformal U(2,2) = Spin(2,4)xU(1) gravity plus conformal structures, based on a generalized MacDowell-Mansouri mechanism, for cosmological gravity of Dark Energy, Dark Matter, and Ordinary matter.

Step 12:

Further:

- a gauge boson emanating from D8 only from its 1E8, iE8, jE8, kE8, and EE8 lattices is a U(5) gauge boson related to Spin(10) and Complex E6.
- a gauge boson emanating from D8 only from its 1E8, iE8, jE8, kE8, EE8, and IE8 lattices is a U (6) gauge boson related to Spin(12) and Quaternionic E7.
- a gauge boson emanating from D8 only from its 1E8, iE8, jE8, kE8, EE8, IE8, and JE8 lattices is a U(7) gauge boson related to Spin(14) and possibly to Sextonionic E(7+(1/2)).
- a gauge boson emanating from D8 only from its 1E8, iE8, jE8, kE8, EE8, IE8, JE8, and KE8 lattices is a U(8) gauge boson related to Spin(16) and Octonionic E8.

These correspondences are based on the natural inclusion of U(N) in Spin(2N) and on Magic Square constructions of the E series of Lie algebras, roughly described as follows:

- 78-dim E6 = 45-dim Adjoint of Spin(10) + 32-dim Spinor of Spin(10) + Imaginary of C;
- 133-dim E7 = 66-dim Adjoint of Spin(12) + 64-dim quaternionic-half-Spinor of Spin(12) + Imaginaries of Q;
- 248-dim E8 = 120-dim Adjoint of Spin(16) + 128-dim half-Spinor of Spin(16)

Physically,

- E6 corresponds to 26-dim String Theory, related to traceless J3(O)o and the symmetric space E6 / F4.
- E7 corresponds to 27-dim M-Theory, related to the Jordan algebra J3(O) and the symmetric space E7 / E6 x U(1).
- E8 corresponds to 28-dim F-Theory, related to the Jordan algebra J4(Q) and the symmetric space E8 / E7 x SU(2).

A Single Cell of the physically realistic 26-dimensional Lattice Bosonic String Theory,

in which Strings are physically interpreted as World-Lines,

can be described by taking the quotient of its 24-dimensional O+, O-, Ov subspace modulo the 24-dimensional Leech lattice, and

its automorphism group is the largest finite sporadic group, the Monster Group M, whose order is

8080, 17424, 79451, 28758, 86459, 90496, 17107, 57005, 75436, 80000, 00000 =

2^46 .3^20 .5^9 .7^6 .11^2 .13^3 .17.19.23.29.31.41.47.59.71

or about 8 x 10^53.

If you use positronium (electron-positron bound state of the two lowest-nonzero-mass Dirac fermions) as a unit of mass Mep = 1 MeV, then it is interesting that the product of the squares of the Planck mass Mpl = 1.2 x 10^22 MeV and W-boson mass Mw = 80,000 MeV gives ((Mpl/Mep)(Mw/Mep))^2 = = 9 x 10^53 which is roughly the Monster order.

The Mpl part of M may be related to Aut(Leech Lattice) = double cover of Co1.

The order of Co1 is 2^21.3^9.5^4.7^2.11.13.23 or about 4 x 10^18.

The Mw part of M may be related to Aut(Golay Code) = M24.

The order of M24 is 2^10.3^3.5.7.11.23 or about 2.4 x 10^8.

If you look at the physically realistic superposition of 8 such Cells, you get 8 copies of the Monster of total order about 6.4 x 10^54, which is roughly the product of the Planck mass and Higgs VEV squared: (1.22 x 10^22)^2 x (2.5 x 10^5)^2 = 9 x 10^54

The full 26-dimensional Lattice Bosonic String Theory can be regarded as an infinite-dimensional Affinization of the Theory of that Single Cell.

James Lepowsky said in math.QA/0706.4072:

"... **the Fischer-Griess Monster M ... was constructed by Griess as a symmetry group (of order about 10^54) of a remarkable new commutative but very, very highly nonassociative, seemingly ad-hoc, algebra B of dimension 196,883**. The "structure constants" of the Griess algebra B were "forced" by expected properties of the conjectured-to-exist Monster. It was proved by J. Tits that M is actually the full symmetry group of B. ...

There should exist a (natural) infinite-dimensional Z-graded module for M (i.e., representation of M)

V = DIRSUM(n=-1,0,1,2,3,...) V_n ...

such that

... the graded dimension of the graded vector space V ... = ... SUM(n=-1,0,1,2,3,...) (dim V_n) q^n

where

J(q) = q^(-1) + 0 + 196884q + higher-order terms,

the classical modular function with its constant term set to 0. J(q) is the suitably normalized generator of the field of SL(2, Z)-modular invariant functions on the upper half-plane, with q = exp(2 pi i tau) , tau in the upper half-plane ...

Conway and Norton conjectured ... for every g in M (not just g = 1), the the generating function

... the graded trace of the action of g on the graded space V ... = ... SUM(n=-1,0,1,2,3,...) (tr g | V_n) q^n

should be the analogous "Hauptmodul" for a suitable discrete subgroup of SL(2,R), a subgroup having a fundamental "genus-zero property," so that its associated field of modular-invariant functions has a single generator (a Hauptmodul) ... (... the graded dimension is of course the graded trace of the identity element g = 1.) The Conway-Norton conjecture subsumed a remarkable coincidence that had been noticed earlier - that **the 15 primes giving rise to the genus-zero property ... are precisely the primes dividing the order of the ... Monster** ...

the McKay-Thompson conjecture ... that there should exist a natural ... infinite-dimensional Z-graded M-module V whose graded dimension is J(q) ... was (constructively) proved .... The graded traces of some, but not all, of the elements of the Monster - the elements of an important subgroup of M, namely, a certain involution centralizer involving the largest Conway sporadic group Co1 - were consequences of the construction, and these graded traces were indeed (suitably) modular functions ... We called this V **"the moonshine module V[flat]"** ...

The **construction ... needed ... a natural infinite-dimensional "affinization" of the Griess algebra B acting on V[flat**]

This "affinization," which was part of the new algebra of vertex operators, is analogous to, but more subtle than, the notion of affine Lie algebra .... More precisely, the vertex operators were needed for a "commutative affinization" of a certain natural 196884-dimensional enlargement B' of B, with an identity element (rather than a "zero" element) adjoined to B. This enlargement B' naturally incorporated the Virasoro algebra - the central extension of the Lie algebra of formal vector fields on the circle - acting on V[flat] ...

The vertex operators were also needed for a natural "lifting" of Griess's action of M from the finite-dimensional space B to the infinite-dimensional structure V[flat], including its algebra of vertex operators and its copy of the affinization of B'.

Thus the Monster was now realized as the symmetry group of a certain explicit "algebra of vertex operators" based on an infinite-dimensional Z-graded structure whose graded dimension is the modular function J(q).

Griess's construction of B and of M acting on B was a crucial guide for us, although we did not start by using his construction; rather, we recovered it, as **a finite-dimensional "slice" of a new infinite-dimensional construction** using vertex operator considerations. ...

The initally strange-seeming finite-dimensional Griess algebra was now embedded in a natural new infinite-dimensional space on which a certain algebra of vertex operators acts ... At the same time, the Monster, a finite group, took on a new appearance by now being understood in terms of a natural infinite-dimensional structure. ... the largest sporadic finite simple group, the Monster, was "really" infinite-dimensional ...

The very-highly-nonassociative Griess algebra, or rather, from our viewpoint, the natural modification of the Griess algebra, with an identity element adjoined, coming from a "forced" copy the Virasoro algebra, became simply the conformal-weight-two subspace of an algebra of vertex operators of a certain "shape." ...

the constant term of J(q) is zero, and this choice of constant term, which is not uniquely determined by number-theoretic principles, is not traditional in number theory. It turned out that the vanishing of the constant term ... was canonically "forced" by the requirement that the Monster should act naturally on V [flat] and on an associated algebra of vertex operators.

This vanishing of the degree-zero subspace of V[flat] is actually analogous in a certain strong sense to the absence of vectors in the Leech lattice of square-length two; the Leech lattice is a distinguished rank-24 even unimodular (self-dual) lattice with no vectors of square-length two.

In addition, this vanishing of the degree-zero subspace of V[flat] and the absence of square-length-two elements of the Leech lattice are in turn analogous to the absence of code-words of weight 4 in the Golay error-correcting code, a distinguished self-dual binary linear code on a 24-element set, with the lengths of all code-words divisible by 4. In fact, the Golay code was used in the original construction of the Leech lattice, and the Leech lattice was used in the construction of V[flat]

This was actually to be expected ... because it was well known that the automorphism groups of both the Golay code and the Leech lattice are (essentially) sporadic finite simple groups; the automorphism group of the Golay code is the Mathieu group M24 and the automorphism group of the Leech lattice is a double cover of the Conway group Co1 mentioned above, and both of these sporadic groups were well known to be involved in the Monster ... in a fundamental way....

The Golay code is actually **unique** subject to its distinguishing properties mentioned above ... and **the Leech lattice is unique** subject to its distinguishing properties mentioned above ... **Is V[flat] unique? If so, unique subject to what? ... this uniqueness is an unsolved problem** ...

V[flat] came to be viewed in retrospect by string theorists as an inherently string-theoretic structure: the "chiral algebra" underlying the Z2-orbifold conformal field theory based on the Leech lattice.

The string-theoretic geometry is this: One takes the torus that is the quotient of 24-dimensional Euclidean space modulo the Leech lattice, and then one takes the quotient of this manifold by the "negation" involution x -> -x, giving rise to an orbit space called an "orbifold"&emdash;a manifold with, in this case, a "conical" singularity. Then one takes the "conformal field theory" (presuming that it exists mathematically) based on this orbifold, and from this one forms a "string theory" in two-dimensional space-time by compactifying a 26-dimensional "bosonic string" on this 24-dimensional orbifold. The string vibrates in a 26-dimensional space, 24 dimensions of which are curled into this 24-dimensional orbifold ...

Borcherds used ... ideas, including his results on generalized Kac-Moody algebras, also called Borcherds algebras, together with certain ideas from string theory, including the "physical space" of a bosonic string along with the "no-ghost theorem" ... to prove the remaining Conway-Norton conjectures for the structure V[flat] ... What had remained to prove was ... that ... the conjugacy classes outside the involution centralizer - were indeed the desired Hauptmoduls ... He accomplished this by constructing a copy of his "Monster Lie algebra" from the "physical space" associated with V[flat] enlarged to a central-charge-26 vertex algebra closely related to the 26-dimensional bosonic-string structure mentioned above. He transported the known action of the Monster from V[flat] to this copy of the Monster Lie algebra, and ... he proved certain recursion formulas ... ... he succeeded in concluding that all the graded traces for V[flat] must coincide with the formal series for the Hauptmoduls ...

this vertex operator algebra V[flat] has the following three simply-stated properties ...

- (1) V[flat], which is an irreducible module for itself ... , is its only irreducible module, up to equivalence ... every module for the vertex operator algebra V [flat] is completely reducible and is in particular a direct sum of copies of itself. Thus the vertex operator algebra V[flat] has no more representation theory than does a field! (I mean a field in the sense of mathematics, not physics. Given a field, every one of its modules - called vector spaces, of course - is completely reducible and is a direct sum of copies of itself.)
- (2) dim V[flat]_0 = 0. This corresponds to the zero constant term of J(q); while the constant term

of the classical modular function is essentially arbitrary, and is chosen to have certain values for certain classical number-theoretic purposes, the constant term must be chosen to be zero for the purposes of moonshine and the moonshine module vertex operator algebra.

- (3) The central charge of the canonical Virasoro algebra in V[flat] is 24. "24" is the "same 24" so basic in number theory, modular function theory, etc. As mentioned above, this occurrence of 24 is also natural from the point of view of string theory.

These three properties are actually "smallness" properties in the sense of conformal field theory and string theory. These properties allow one to say that V[flat] essentially defines the smallest possible nontrivial string theory ... (These "smallness" properties essentially amount to: "no nontrivial representation theory," "no nontrivial gauge group," i.e., "no continuous symmetry," and "no nontrivial monodromy"; this last condition actually refers to both the first and third "smallness" properties.)

Conversely, conjecturally ... V[flat] is the unique vertex operator algebra with these three "smallness" properties (up to isomorphism). This conjecture ... remains unproved. It would be the conformal-field-theoretic analogue of the uniqueness of the Leech lattice in sphere-packing theory and of the uniqueness of the Golay code in error-correcting code theory ...

Proving this uniqueness conjecture can be thought of as the "zeroth step" in the program of classification of (reasonable classes of) conformal field theories. M. Tuite has related this conjecture to the genus-zero property in the formulation of monstrous moonshine.

Up to this conjecture, then, we have the following remarkable characterization of the largest sporadic finite simple group: **The Monster is the automorphism group of the smallest nontrival string theory** that nature allows ... **Bosonic 26-dimensional space-time ... "compactified" on 24 dimensions, using the orbifold construction V[flat]** ... or more precisely, the automorphism group of the vertex operator algebra with the canonical "smallness" properties. ...

This definition of the Monster in terms of "smallness" properties of a vertex operator algebra provides a remarkable motivation for the definition of the precise notion of vertex (operator) algebra. The discovery of string theory (as a mathematical, even if not necessarily physical) structure sooner or later must lead naturally to the question of whether this "smallest" possible nontrivial vertex operator algebra V . exists, and the question of what its symmetry group (which turns out to be the largest sproradic finite simple group) is.

And on the other hand, the classification of the the finite simple groups - a mathematical problem of the absolutely purest possible sort - leads naturally to the question of what natural structure the largest sporadic group is the symmetry group of; the answer entails the development of string theory and vertex operator algebra theory (and involves modular function theory and monstrous moonshine as well).

The Monster, a singularly exceptional structure - in the same spirit that the Lie algebra E8 is "exceptional," though M is far more "exceptional" than E8 - helped lead to, and helps shape, the very general theory of vertex operator algebras. (The exceptional nature of structures such as E8, the Golay code and the Leech lattice in fact played crucial roles in the construction of V[flat] ...

V[flat] is defined over the field of real numbers, and in fact over the field of rational numbers, in such a way that the Monster preserves the real and in fact rational structure, and that the Monster preserves a rational-valued positive-definite symmetric bilinear form on this rational structure. ...

the "orbifold" construction of V[flat] ...[has been]... interpreted in terms of algebraic quantum field theory, specifically, in terms of local conformal nets of von Neumann algebras on the circle ...

the notion of vertex operator algebra is actually the "one-complex-dimensional analogue" of the notion of Lie algebra. But at the same time that it is the "one-complex-dimensional analogue" of the notion of Lie algebra, the notion of vertex operator algebra is also the "one-complex- dimensional analogue" of the notion of commutative associative algebra (which again is the corresponding "one-real-dimensional" notion). ... This analogy with the notion of commutative associative algebra comes from the "commutativity" and "associativity" properties of the vertex operators ... in a vertex operator algebra ...

The remarkable and paradoxical-sounding fact that the notion of vertex operator algebra can be, and is, the "one-complex-dimensional analogue" of BOTH the notion of Lie algebra AND the notion of commutative associative algebra lies behind much of the richness of the whole theory, and of string theory and conformal field theory.

When mathematicians realized a long time ago that complex analysis was qualitatively entirely different from real analysis (because of the uniqueness of analytic continuation, etc., etc.), a whole new point of view became possible. In vertex operator algebra theory and string theory, there is again a fundamental passage from "real" to "complex," this time leading from the concepts of both Lie algebra and commutative associative algebra to the concept of vertex operator algebra and to its theory, and also leading from point particle theory to string theory. ...

While a string sweeps out a two-dimensional (or, as we've been mentioning, one-complex-dimensional) "worldsheet" in space-time, **a point particle of course sweeps out a one-real-dimensional "world-line"** in space-time, with time playing the role of the "one real dimension," and this "one real dimension" is related in spirit to the "one real dimension" of the classical operads that I've briefly referred to - the classical operads "mediating" the notion of associative algebra and also the notion of Lie algebra (and indeed, any "classical" algebraic notion), and in addition "mediating" the classical notion of braided tensor category. The "sequence of operations performed one after the other" is related (not perfectly, but at least in spirit) to the ordering ("time-ordering") of the real line.

But as we have emphasized, the "algebra" of vertex operator algebra theory and also of its representation theory (vertex tensor categories, etc.) is "mediated" by an (essentially) one-complex-dimensional (analytic partial) operad (or more precisely, as we have mentioned, the infinite-dimensional analytic structure built on this). When one needs to compose vertex operators, or more generally, intertwining operators, after the formal variables are specialized to complex variables, one must choose not merely a (time-)ordered sequencing of them, but instead, a suitable complex number, or more generally, an analytic local coordinate as well, for each of the vertex operators.

This process, very familiar in string theory and conformal field theory, is a reflection of how the one-complex-dimensional operadic structure "mediates" the algebraic operations in vertex operator algebra theory.

Correspondingly, "algebraic" operations in this theory are not instrinsically "time-ordered"; they are instead controlled intrinsically by the one-complex-dimensional operadic structure. The "algebra" becomes intrinsically geometric.

"Time," or more precisely, as we discussed above, the one-real-dimensional world-line, is being replaced by a one-complex-dimensional world-sheet.

This is the case, too, for the vertex tensor category structure on suitable module categories. In vertex operator algebra theory, "algebra" is more concerned with one-complex-dimensional geometry than with one-real-dimensional time. ...".
