

E6, Strings, Branes, and the Standard Model

- [E6 String Structure](#)
- [Standard Model String Theory](#)

In his paper [hep-th/0112261](#) entitled **Algebraic Dreams**, Pierre Ramond says:

"... Nature shows that space-time symmetries with dynamics associated with gravity, and internal symmetries with their dynamics described by Yang-Mills theories, can coexist peacefully. How does She do it? ... there remain important unanswered questions. ...".

According to [a superstring theory web site](#):

"... For bosonic strings ...[you]... can ... do quantum mechanics sensibly only if the spacetime dimensions number 26. For superstrings we can whittle it down to 10. ...

A Brief Table of String Theories		
Type	Spacetime Dimensions	Details
Bosonic	26	Only bosons, no fermions means only forces, no matter, with both open and closed strings. Major flaw: a particle with imaginary mass, called the tachyon
I	10	Supersymmetry between forces and matter, with both open and closed strings, no tachyon, group symmetry is SO(32)
IIA	10	Supersymmetry between forces and matter, with closed strings only, no tachyon, massless fermions spin both ways (nonchiral)
IIB	10	Supersymmetry between forces and matter, with closed strings only, no tachyon, massless fermions only spin one way (chiral)

HO	10	Supersymmetry between forces and matter, with closed strings only, no tachyon, heterotic, meaning right moving and left moving strings differ, group symmetry is SO(32)
HE	10	Supersymmetry between forces and matter, with closed strings only, no tachyon, heterotic, meaning right moving and left moving strings differ, group symmetry is E₈ x E₈

... There are higher dimensional objects in string theory with dimensions from zero (points) to nine, called p-branes. In terms of branes, what we usually call a membrane would be a two-brane, a string is called a one-brane and a point is called a zero-brane. ... A special class of p-branes in string theory are called D branes. Roughly speaking, a D brane is a p-brane where the ends of open strings are localized on the brane. A D brane is like a collective excitation of strings. ...

... the five superstring theories are connected to one another as if they are each a special case of some more fundamental theory ...

... an eleven dimensional theory of supergravity, which is supersymmetry combined with gravity ... didn't work as a unified theory of particle physics, because it doesn't have a sensible quantum limit as a point particle theory. But this eleven dimensional theory ... came back to life in the strong coupling limit of superstring theory in ten dimensions ... M theory is the unknown eleven-dimensional theory whose low energy limit is the supergravity theory in eleven dimensions ... many people have taken to also using M theory to label the unknown theory believed to be the fundamental theory from which the known superstring theories emerge as special limits ...

... **We still don't know the fundamental M theory ...**".

The purpose of this paper is to give an example of

Algebraic Reality:

A String Theory with E6 Structure that accurately represents Gravity and the Standard Model.

The E6 exceptional Lie algebra string theory is a counterexample to Pierre Ramond's [statement](#):

"... M-theory and Superstring theories ... are the only examples of theories where ... union ...[of]... gravity ... and internal symmetries ... appears possible ...",

but is consistent with Pierre Ramond's [statement](#):

"... **Nature relishes unique mathematical structures. ... The Exceptional Algebras are most unique and beautiful among [Lie Algebras](#)**, and no one should be surprised if Nature uses them. ...".

Although Raymond sees the tensor-spinor relationships of exceptional groups as an obstacle, [saying](#)

"... The use of exceptional groups to describe space-time symmetries has not been as fruitful [as the use of classical groups] ... One obstacle has been that exceptional algebras relate tensor and [spinor](#) representations of their orthogonal subgroups, while [Spin Statistics](#) requires them to be treated differently. ...",

I see the exceptional tensor-spinor relationships of E6 as a way to introduce fermions into String Theory [without naive 1-1 fermion-boson supersymmetry](#).

[Ramond accurately describes E6 in these terms](#):

"... **The traceless [Jordan matrices \[J3\(O\)o \]](#) ... (3x3) traceless octonionic hermitian matrices, each labelled by 26 real parameters ... span the 26 representation of [the 52-dimensional exceptional Lie algebra F4]. One can supplement the F4 transformations by an additional 26 parameters ... leading to a group with 78 parameters. These extra transformations are non-compact, and close on the F4 transformations, **leading to the exceptional group E6(-26)**. The subscript in parenthesis denotes the number of non-compact minus the number of compact generators. ...".**

The following is my proposal to use the exceptional Lie algebra E6(-26), which I will for the rest of this message write as E6, to introduce fermions into string theory in a new way, based on the exceptional E6 relations between bosonic vectors/bivectors and fermionic spinors, in which 16 of the 26 dimensions are seen as orbifolds whose 8 + 8 singularities represent first-generation fermion particles and antiparticles.

This structure allows string theory to be physically interpreted as a theory of interaction among world-lines in the Many-Worlds.

According to Soji Kaneyuki, in Graded Lie Algebras, Related Geometric Structures, and Pseudo-hermitian Symmetric Spaces, Analysis and Geometry on Complex Homogeneous Domains, by Jacques Faraut, Soji Kaneyuki, Adam Koranyi, Qi-keng Lu, and Guy Roos (Birkhauser 2000), E6 as a [Graded Lie Algebra](#) with 5

grades:

$$g = E6 = g(-2) + g(-1) + g(0) + g(1) + g(2)$$

such that

- $g(0) = so(8) + R + R$
- $\dim_{\mathbb{R}} g(-1) = \dim_{\mathbb{R}} g(1) = 16 = 8 + 8$
- $\dim_{\mathbb{R}} g(-2) = \dim_{\mathbb{R}} g(2) = 8$

Here, step-by-step, is a description of the E6 structure:

Step 1:

$g(0) = so(8)$		28 gauge bosons
	$+ R + R$	
$\dim_{\mathbb{R}} g(-1) = \dim_{\mathbb{R}} g(1) = 16 = 8 + 8$		26-dim string spacetime with $J3(0)$ structure
$\dim_{\mathbb{R}} g(-2) = \dim_{\mathbb{R}} g(2) = 8$		

Step 2:

The E6 GLA has an Even Subalgebra g_E (Bosonic) and an Odd Part g_O (Fermionic):

BOSONIC $g_E = g(-2) + g(0) + g(2)$

FERMIONIC $g_O = g(-1) + g(1)$

Step 3:

BOSONIC

$g(0) = so(8)$	28 gauge bosons
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$$\dim_{\mathbb{R}} \mathfrak{g}(-2) = \dim_{\mathbb{R}} \mathfrak{g}(2) = 8 \quad \begin{array}{c} - \\ | \\ - \end{array} \quad \begin{array}{l} + R + R \\ 10\text{-dim spacetime} \end{array}$$

FERMIONIC

$$\dim_{\mathbb{R}} \mathfrak{g}(-1) = \dim_{\mathbb{R}} \mathfrak{g}(1) = 16 = 8 \quad \begin{array}{l} 8\text{-dim orbifold} \\ + \\ 8 \\ 8\text{-dim orbifold} \end{array}$$

Giving the Fermionic sector [orbifold](#) structure gives each point of the string/world-line a discrete value corresponding to one of the $8+8 = 16$ fundamental first-generation fermion particles or antiparticles.

Step 4:

BOSONIC

$$\mathfrak{g}(0) = \mathfrak{so}(8) \quad 28 \text{ gauge bosons}$$

$$\dim_{\mathbb{R}} \mathfrak{g}(-2) = \dim_{\mathbb{R}} \mathfrak{g}(2) = 8 \quad \begin{array}{c} - \\ | \\ - \end{array} \quad \begin{array}{l} + R + R \\ 10\text{-dim spacetime} \end{array}$$

FERMIONIC

$$\dim_{\mathbb{R}} \mathfrak{g}(-1) = \dim_{\mathbb{R}} \mathfrak{g}(1) = 16 = 8 \quad \begin{array}{l} 8 \text{ fermions} \\ + \\ 8 \\ 8 \text{ antifermions} \end{array}$$

Step 5:

BOSONIC

$$\mathfrak{g}(0) = \mathfrak{so}(8) \quad \begin{array}{l} 16\text{-dim conformal } U(2,2) \\ + \\ 12\text{-dim } SU(3) \times SU(2) \times U(1) \end{array}$$

$$\dim_{\mathbb{R}} \mathfrak{g}(-2) = \dim_{\mathbb{R}} \mathfrak{g}(2) = 4 \quad \begin{array}{c} - \\ | \\ - \end{array} \quad \begin{array}{l} + R + R \\ 6\text{-dim conformal spacetime} \end{array}$$

$$+ \quad - |$$

$$4 \quad 4\text{-dim internal symmetry space}$$

FERMIONIC

$$\dim_{\mathbb{R}} \mathfrak{g}(-1) = \dim_{\mathbb{R}} \mathfrak{g}(1) = 16 = 8 \quad 8 \text{ fermions (3 gen)}$$

$$+ 8 \quad 8 \text{ antifermions (3 gen)}$$

[Dimensional reduction of spacetime breaks so\(8\) to U\(2,2\) and SU\(3\)xSU\(2\)xU\(1\) and also introduces 3 generations of fermion particles and antiparticles.](#)

Step 6:

BOSONIC

$$\mathfrak{g}(0) = \mathfrak{so}(8) \quad 16\text{-dim conformal } U(2,2)$$

$$+ \quad 12\text{-dim } SU(3) \times SU(2) \times U(1)$$

$$+ \mathbb{R} + \mathbb{R} \quad 2 \text{ spacetime conformal dim}$$

$$\dim_{\mathbb{R}} \mathfrak{g}(-2) = \dim_{\mathbb{R}} \mathfrak{g}(2) = 4 \quad 4\text{-dim physical spacetime}$$

$$+ \quad 4 \quad 4\text{-dim internal symmetry space}$$

FERMIONIC

$$\dim_{\mathbb{R}} \mathfrak{g}(-1) = \dim_{\mathbb{R}} \mathfrak{g}(1) = 16 = 8 \quad 8 \text{ fermions (3 gen)}$$

$$+ 8 \quad 8 \text{ antifermions (3 gen)}$$

The 2 spacetime [conformal](#) dimensions $\mathbb{R}+\mathbb{R}$ are related to complex structure of

- spacetime $\mathfrak{g}(-2) + \mathfrak{g}(2)$ and
- fermionic $\mathfrak{g}(-1) + \mathfrak{g}(1)$.

[Physical spacetime and internal symmetry space](#), and [fermionic representation spaces](#), are related to [Shilov boundaries of the corresponding complex domains](#).

The E6 String Structure described above allows construction of a Realistic String Theory:

This construction was motivated by a March 2004 [sci.physics.research thread Re: photons from strings?](#) in which John Baez asked:

"... has anyone figured out a way to ... start with string theory ... to get just photons on Minkowski spacetime ..." ?

Lubos Motl noted "... **string theory always contains gravity** ... Gravity is always contained as a vibration of a closed string, and closed strings can always be created from open strings....".

Urs Schreiber said "... the low energy effective worldsheet theory of a single flat **D3 brane of the bosonic string** is, to lowest nontrivial order, just U(1) gauge theory in 4D ...".

Aaron Bergman noted "... there are a bunch of scalars describing the transverse fluctuations of the brane ...".

Urs Schreiber said "... I guess that's why you have to **put the brane at the singularity of an orbifold if you want to get rid of the scalars** ... if the number of dimensions is not an issue the simplest thing probably would be to consider the single space-filling D25 brane of the bosonic string. This one does not have any transverse fluctuations and there is indeed only the U(1) gauge field ...".

Aaron Bergman replied "... Unfortunately, there's a tadpole in that configuration. You need 8192 D25 branes to cancel it. ...".

Lubos Motl pointed out the existence of brane structures other than massless vectors, saying "... A D-brane contains other massless states, e.g. the **transverse scalars** (and their **fermionic superpartners**). It also contains an **infinite tower of excited massive states**. Finally, a D-brane in the full string theory is coupled to **the bulk which inevitably contains gravity as well as other fields and particles**. ... N coincident D-branes carry a U(N) gauge symmetry (and contain the appropriate gauge N^2 bosons, as you explained). Moreover, if this stack of N D-branes approaches **an orientifold**, they meet their mirror images and U(N) is extended to O(2N) or USp(2N). The brane intersections also carry **new types of matter - made of the open strings stretched from one type of brane to the other** - but these new fields are **not** gauge fields, and they don't lead to new gauge symmetries. For example, there are **scalars whose condensation is able to join two intersecting D2-branes into a smooth, connected,**

hyperbolically shaped objects (D2-branes). ... the number of D-branes can be determined or bounded by anomaly cancellation and similar requirements. For example, the spacetime filling D9-branes in type I theory must generate the $SO(32)$ gauge group, otherwise the theory is anomalous. (There are other arguments for this choice of 16+16 branes, too.)..."

What follows on this page is my construction of

a specific example of a String Theory with [E6 structure](#) containing gravity and the $U(1)\times SU(2)\times SU(3)$ Standard Model.

As to how my simple model is affected by some of the complications mentioned by Lubos Motl:

- Transverse scalars are taken care of by Orbifolding as suggested by Urs Schreiber.
- Fermionic superpartners are taken care of by not using naive 1-1 fermion-boson supersymmetry.
- The infinite tower of excited massive states is related to Regge trajectories which in turn are related to [interactions among strings considered as world-lines in the Many-Worlds](#).
- Bulk gravity is included.
- There are no orientifolds.
- Open strings from one brane to another, as vacuum loops, look like exchange of closed loops and are related to gravity among branes and [the Bohm-type quantum potential](#).
- Scalar condensates are related to Dilatons which in turn are related to [interactions among strings considered as world-lines in the Many-Worlds](#).
- I have not fully investigated all potential anomaly problems.

Further, string theory Tachyons are related to [interactions among strings considered as world-lines in the Many-Worlds](#).

In short,

the complications are either taken care of in the construction of the model or are useful in describing the [Bohm-type quantum potential interactions among strings considered as world-lines in the Many-Worlds](#).

Here is some further background, from Joseph Polchinski's book String Theory vol. 1 (Cambridge 1998), in Chapter 8 and the Glossary:

"... a ... D-brane ...[is]... a dynamical object ... a flat hyperplane ...[for which]... a certain open string state corresponds to fluctuation of its shape ...

... A D25-brane fills space, so the string endpoint can be anywhere ...

... When no D-branes coincide there is just one massless vector on each, giving the gauge group $U(1)^n$ in all.

If r D-branes coincide, there are new massless states because strings that are stretched between these branes can have vanishing length: ... Thus, there are r^2 vectors, forming the adjoint of a $U(r)$ gauge group. ... there will also be r^2 massless scalars from the components normal to the D-brane. ...

... The massless fields on the world-volume of a D_p -brane are a $U(1)$ vector plus $25 - p$ world-brane scalars describing the fluctuations. ... The fields on the brane are the embedding $X^u(x)$ and the gauge field $A_a(x)$...

... For n separated D-branes, the action is n copies of the action for a single D-brane. ... when the D-branes are coincident there are n^2 rather than n massless vectors and scalars on the brane ...

... The fields $X^u(x)$ and $A_a(x)$ will now be $n \times n$ matrices ...

... the gauge field ... becomes a non-Abelian $U(n)$ gauge field ...

... the collective coordinates ... X^u ... for the embedding of n D-branes in spacetime are now enlarged to $n \times n$ matrices. This 'noncommutative geometry' ...[may be]... an important hint about the nature of spacetime. ...

...[an]... orbifold ...(*noun*)...[is]... a coset space M/H , where H is a group of discrete symmetries of a manifold M . The coset is singular at the fixed points of H ...(*verb*)...[is]... to produce such a ... string theory by gauging H ...

... To determine the actual value of the D-brane tension ... Consider two parallel D_p -branes ...[They]... can feel each other's presence by exchanging closed strings ...[which is equivalent to]... a vacuum loop of an open string with one end on each D-brane ... The ... analogous ... field theory graph ... is the exchange of a single graviton or dilaton between the D-branes....".

Here, step-by-step, is the String/Brane construction:

Step 1:

Consider the [26 Dimensions](#) of [String Theory](#) as the 26-dimensional traceless part $J_3(O)_o$

$$\begin{array}{ccc}
 a & O_+ & O_V \\
 O_+^* & b & O_- \\
 O_V^* & O_-^* & -a-b
 \end{array}$$

(where O_V , O_+ , and O_- are in Octonion space with basis $\{1, i, j, k, E, I, J, K\}$ and a and b are real numbers with basis $\{1\}$)

of the 27-dimensional [Jordan algebra](#) $J_3(O)$ of [3x3 Hermitian Octonion matrices](#).

Step 2:

Take Urs Schreiber's **D3 brane** to correspond to the **Imaginary Quaternionic associative subspace** spanned by $\{i, j, k\}$ in the 8-dimensional Octonionic O_V space.

Step 3:

Compactify the 4-dimensional co-associative subspace spanned by $\{E, I, J, K\}$ in the Octonionic O_V space as a $CP^2 = SU(3)/U(2)$, with its 4 world-brane scalars corresponding to the 4 covariant components of a Higgs scalar.

Add this subspace to D_3 , to get D_7 .

Step 4:

[Orbifold](#) the 1-dimensional Real subspace spanned by $\{1\}$ in the Octonionic O_V space by the discrete multiplicative group $Z_2 = \{-1, +1\}$, with its fixed points $\{-1, +1\}$ corresponding to past and future time. This discretizes time steps and gets rid of the world-brane scalar corresponding to the subspace spanned by $\{1\}$ in O_V . It also gives our brane a 2-level timelike structure, so that its past can connect to the future of a preceding brane and its future can connect to the past of a succeeding brane.

Add this subspace to D7, to get D8.

D8, our basic Brane, looks like two layers (past and future) of D7s.

Beyond D8 our String Theory has $26 - 8 = 18$ dimensions, of which $25 - 8$ have corresponding world-brane scalars:

- 8 world-brane scalars for Octonionic O+ space;
- 8 world-brane scalars for Octonionic O- space;
- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 5:

To use Urs Schreiber's idea to get rid of the world-brane scalars corresponding to the Octonionic O+ space, orbifold it by the 16-element discrete multiplicative group $\text{Oct}16 = \{+/-1, +/-i, +/-j, +/-k, +/-E, +/-I, +/-J, +/-K\}$ to reduce O+ to 16 singular points $\{-1, -i, -j, -k, -E, -I, -J, -K, +1, +i, +j, +k, +E, +I, +J, +K\}$.

- Let the 8 O+ singular points $\{-1, -i, -j, -k, -E, -I, -J, -K\}$ correspond to the fundamental fermion particles {neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark} located on the past D7 layer of D8.
- Let the 8 O+ singular points $\{+1, +i, +j, +k, +E, +I, +J, +K\}$ correspond to the fundamental fermion particles {neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark} located on the future D7 layer of D8.

This gets rid of the 8 world-brane scalars corresponding to O+, and leaves:

- 8 world-brane scalars for Octonionic O- space;
- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 6:

To use Urs Schreiber's idea to get rid of the world-brane scalars corresponding to the Octonionic O- space, orbifold it by the 16-element discrete multiplicative group $\text{Oct}16 = \{+/-1, +/-i, +/-j, +/-k, +/-E, +/-I, +/-J, +/-K\}$ to

reduce O^- to 16 singular points $\{-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K\}$.

- Let the 8 O^- singular points $\{-1,-i,-j,-k,-E,-I,-J,-K\}$ correspond to the fundamental fermion anti-particles {anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark} located on the past D7 layer of D8.
- Let the 8 O^- singular points $\{+1,+i,+j,+k,+E,+I,+J,+K\}$ correspond to the fundamental fermion anti-particles {anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark} located on the future D7 layer of D8.

This gets rid of the 8 world-brane scalars corresponding to O^- , and leaves:

- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

[Here is some discussion of some symmetries of fermion particles and antiparticles.](#)

Step 7:

Let the 1 world-brane scalar for real a space correspond to a Bohm-type Quantum Potential acting on strings in the **stack of D8 branes**.

Interpret strings as world-lines in the Many-Worlds, short strings representing virtual particles and loops.

Step 8:

Fundamentally, [physics is described on HyperDiamond Lattice structures](#).

[There are 7 independent E8 lattices](#), each corresponding to one of the 7 imaginary octonions. They can be described as $iE8$, $jE8$, $kE8$, $EE8$, $IE8$, $JE8$, and $KE8$.

Further, [an 8th naturally related, but dependent, E8 lattice](#) corresponds to the real octonions and can be described as $1E8$.

Give each D8 brane structure based on Planck-scale E8 lattices so that **each D8 brane is a superposition/intersection/coincidence of the eight E8 lattices**.

Step 9:

Since Polchinski says "... If r D-branes coincide ... there are r^2 vectors, forming the adjoint of a $U(r)$ gauge group ...", make the following assignments:

- a gauge boson emanating from D8 only from its 1E8 lattice is a $U(1)$ photon;
- a gauge boson emanating from D8 only from its 1E8 and EE8 lattices is a $U(2)$ weak boson;
- a gauge boson emanating from D8 only from its IE8, JE8, and KE8 lattices is a $U(3)$ gluon.

Note that I do not consider it problematic to have $U(2)$ and $U(3)$ instead of $SU(2)$ and $SU(3)$ for the weak and color forces, respectively. [Here is some further discussion of the global Standard Model group structure.](#) [Here is some discussion of the root vector structures of the Standard Model groups.](#)

Step 10:

Since Polchinski says "... there will also be r^2 massless scalars from the components normal to the D-brane. ... the collective coordinates ... X^u ... for the embedding of n D-branes in spacetime are now enlarged to $n \times n$ matrices. This 'noncommutative geometry' ...[may be]... an important hint about the nature of spacetime. ...", make the following assignment:

The 8×8 matrices for the collective coordinates linking a D8 brane to the next D8 brane in the stack are needed to connect

the eight E8 lattices of the D8 brane

to the eight E8 lattices of the next D8 brane in the stack.

We have now accounted for all the scalars, and, since, as Lubos Motl noted, "... string theory always contains gravity ...",

we have here at Step 10 **a specific example of a String Theory containing gravity and the $U(1) \times SU(2) \times SU(3)$ Standard Model.**

Step 11:

We can go a bit further by noting that we have not described gauge bosons emanating from D8 from its iE8, jE8, or kE8 lattices. Therefore, make the following assignment:

- a gauge boson emanating from D8 only from its 1E8, iE8, jE8, and kE8 lattices is a U(2,2) conformal gauge boson.

We have here at Step 10 a String Theory containing the Standard Model plus two forms of gravity:

- closed-string gravity and
- conformal $U(2,2) = Spin(2,4) \times U(1)$ gravity plus conformal structures, based on [a generalized MacDowell-Mansouri mechanism](#).

I conjecture that those two forms of gravity are not only consistent, but that the structures of each will shed light on the structures of the other, and that the conformal structures are related to [the conformal gravity ideas of I. E. Segal](#).

Step 12:

Going a bit further leads to consideration of the exceptional E-series of Lie algebras, as follows:

a gauge boson emanating from D8 only from its 1E8, iE8, jE8, kE8, and EE8 lattices is a U(5) gauge boson related to Spin(10) and Complex E6.

a gauge boson emanating from D8 only from its 1E8, iE8, jE8, kE8, EE8, and IE8 lattices is a U(6) gauge boson related to Spin(12) and Quaternionic E7.

a gauge boson emanating from D8 only from its 1E8, iE8, jE8, kE8, EE8, IE8, and JE8 lattices is a U(7) gauge boson related to Spin(14) and possibly to Sextonionic $E(7+(1/2))$.

a gauge boson emanating from D8 only from its 1E8, iE8, jE8, kE8, EE8, IE8, JE8, and KE8 lattices is a U(8) gauge boson related to Spin(16) and Octonionic E8.

These correspondences are based on the natural inclusion of U(N) in Spin(2N) and on [Magic Square constructions](#) of the E series of [Lie algebras](#), roughly described as follows:

- 78-dim E6 = 45-dim Adjoint of Spin(10) + 32-dim Spinor of Spin(10) + Imaginary of C;
- 133-dim E7 = 66-dim Adjoint of Spin(12) + 64-dim Spinor of Spin(12) + Imaginaries of Q;
- 248-dim E8 = 120-dim Adjoint of Spin(16) + 128-dim half-Spinor of Spin(16)

Physically,

- E6 corresponds to 26-dim String Theory, related to traceless $J_3(O)$ and the symmetric space E_6 / F_4 .
- E7 corresponds to 27-dim M-Theory, related to the Jordan algebra $J_3(O)$ and the symmetric space E_7

/ $E_6 \times U(1)$.

- E_8 corresponds to 28-dim F-Theory, related to the Jordan algebra $J_4(\mathbb{Q})$ and the symmetric space $E_8 / E_7 \times SU(2)$.

Note on Sextonions:

I am not yet clear about how the Sextonionic $E(7+(1/2))$ works. It was only recently developed by J. M. Landsberg and Laurent Manivel in their paper "The sextonions and $E_{7\frac{1}{2}}$ " at math.RT/0402157. Of course, the Sextonion algebra is not a real division algebra, but it does have interesting structure. In their paper, Landsberg and Manivel say:

"... We fill in the "hole" in the exceptional series of Lie algebras that was observed by Cvitanovic, Deligne, Cohen and deMan. More precisely, we show that the intermediate Lie algebra between E_7 and E_8 satisfies some of the decomposition and dimension formulas of the exceptional simple Lie algebras. A key role is played by the sextonions, a six dimensional algebra between the quaternions and octonions. Using the sextonions, we show simliar results hold for the rows of an expanded Freudenthal magic chart. We also obtain new interpretations of the adjoint variety of the exceptional group G_2

... the orthogonal space to a null-plane U , being equal to the kernel of a rank-two derivation, is a six-dimensional subalgebra of O

... The decomposition ... into the direct sum of two null-planes, is unique. ...[this]... provides an interesting way to parametrize the set of quaternionic subalgebras of O".

Some possibly related facts of which I am aware include:

- The set of Quaternionic subalgebras of Octonions = $SU(3) = G_2 / Spin(4)$.
- $G_2 / SU(3) = S_6$ is almost complex but not complex and is not Kaehler. Its almost complex structure is not integrable. See chapter V of Curvature and Homology, rev. ed., by Samuel I. Goldberg (Dover 1998).
- It may be that the sextonions and S_6 are related to $Spin(4)$ as the 6-dim conformal vector space of $SU(2,2) = Spin(2,4)$ is related to 4-dim Minkowski space.

Note on [the Monster](#):

The 26 dimensions of String Theory might be related to the 26 Sporadic Finite Simple Groups, the largest of which, [the Monster](#), has about 8×10^{53} elements. If you use positronium (electron-positron bound state of the two lowest-nonzero-mass Dirac fermions) as a unit of mass $M_{ep} = 1$ MeV, then it is interesting that the product of the squares of the Planck mass $M_{pl} = 1.2 \times 10^{22}$ MeV and W-boson mass $M_w = 80,000$ MeV gives $(M_{pl}/M_{ep})(M_w/M_{ep})^2 = 9 \times 10^{53}$ which is roughly the Monster order. Maybe the Monster shows how, in the world of particle physics, "**big**" things like [Planck mass](#) and [W-bosons](#) are related to "**little**" (but not zero-mass) things like [electrons and positrons](#), thus giving you some persepective on the world of fundamental particles.

[Tony's Home Page](#) This paper is also at [CERN-CDS-EXT-2004-031](#).